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Aspects of Multi-Junction AC Josephson Effect in Superconductor

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Abstract

A model of the multi-junction AC Josephson effect in superconductor has been proposed recently and showed that the amplitude of the resultant current for multi-junction increases with the increase of number of junctions. For maximum current, the phase and frequency follow the relation $\omega_J t+\delta_N=(4n+1)\pi/2$. Beside this, it has also been shown that in the absence of applied voltage across the two junctions, the device is similar to the DC SQUID. But there is a question, what is the physical reason that the current increase with the increasing number of junctions? Here, we give a satisfactory explanation of the problem mentioned above introducing the concept of dynamic resistance of the multi-Josephson junction. The numerical result indicate that the dynamic resistance decreases with the increase of number of junctions and hence the amplitude of current increases. Finally, the possible technology based on this model is indicated.

Keywords: Ac Josephson effect, Dynamic resistance, Superconducting magnetic energy storage (SMES), Electromagnetic rail launcher (EMRL).

1. Introduction

The Josephson effect [1-4] is an interesting phenomenon in superconductivity and opened a broad avenue of research in science and technology. Many devices, like SQUID (consist of one or two Josephson junctions), high-frequency oscillator, voltage standards, ultrafast switching computer elements, etc., have been developed based on this effect. The ac Josephson effect (the effect due to Josephson junction in presence of applied voltage) is responsible for these types of superconducting devices. That is why we are interested to know what happens if the number of Josephson junction is increased. For avoiding complexity, we have studied the parallel combination of the Josephson junction. For a parallel connection, it is very easy to add or fabricate a number of junctions in the presence of applied voltage and the resultant current will be the vector sum of the current in each junction. We use simple mathematics for the junctions connected in parallel and constitute a model of the multi-junction AC Josephson effect. In section-2, we give a brief account of the previous model first and then its physical interpretation using the concept of dynamic resistance. Numerical result is given in section-3. In section-4, the possible applications based on this model in superconductor technology have been indicated. Finally, the discussion and conclusion is given in section-5.

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2. Theoretical Model

2.1 Multi-junction ac Josephson effect in superconductor

Let us consider two Josephson junctions (I and II) connected in parallel at the point 'a' and 'b', making a superconducting loop L. Let there is a constant DC voltage (V_O) across both the junctions. The circuit connection is shown in Figure 1.

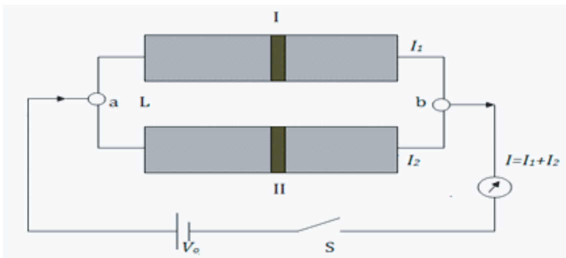

Fig. 1: Schematic diagram of two Josephson junctions connected in parallel with a constant DC voltage source (V_O).

Since the connection is parallel the voltage is same across both the junctions. But different current pass

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through the junctions I and II. Now the current equations for each junction [1-3] in presence of DC voltage (V_O) is written as

$$I_1 = I_{o1} \sin(\omega_J t + \delta_1) \quad (1)$$

and

$$I_2 = I_{o2} \sin(\omega_J t + \delta_2) \quad (2)$$

Here, I_{01} and I_{02} be the maximum current flowing through the junction I and II; $\delta_1 = \theta_{I1} \sim \theta_{I2}$ and $\delta_2 = \theta_{II1} \sim \theta_{II2}$ be the phase differences across the junction I and II, respectively. For each junction, $\omega_J = 2eV_o/\hbar$ epresents the Josephson frequency of oscillation of the ac current.

After circuit connection (shown in Figure 1), the resultant (total) current flowing through the points 'a' and 'b' is the vector some of the current in each junction:

$$\begin{aligned} I_{total} &= I_1 + I_2 \\ &= I_{01} \sin(\omega_J t + \delta_1) + I_{02} \sin(\omega_J t + \delta_2) \\ &= I_{00} \sin(\omega_J t + \varphi). \end{aligned} \quad (3)$$

Here, $$\varphi = \tan^{-1} \frac{I_{o1} \sin\delta_1 + I_{o2} \sin\delta_2}{I_{o1} \cos\delta_1 + I_{o2} \cos\delta_2}, \quad (4)$$

$$I_{00} = \sqrt{I_{o1}^{\ 2} + I_{o2}^{\ 2} + 2I_{o1} I_{o2} \cos(\delta_1 - \delta_2)}. \quad (5)$$

The Eq.(3) is similar to the AC Josephson current, i.e., current in Eq. (1) and (2), but maximum current and phase differences are different. For identical Josephson junctions, we may assume that I_{01} and I_{02}. Using some mathematical algebra, we have

$$\varphi = \frac{\delta_1 + \delta_2}{2}. \quad (6)$$

This equation represents the total phase difference across both the junctions in the presence of dc voltage (V_O).

Using the similar argument, we also have

$$I_{00} = 2I_0 \cos\left(\frac{\delta_1 - \delta_2}{2}\right). \quad (7)$$

Using equations (6) and (7) in equation (3), the total result current for two Josephson junction in presence of DC voltage is, therefore, written as

$$I_{total} = 2I_o \cos\left(\frac{\delta_1 - \delta_2}{2}\right) \sin\left[\omega_J t + \left(\frac{\delta_1 + \delta_2}{2}\right)\right]. \quad (8)$$

This is *AC Josephson effect for two identical Josephson junction.*

If we assume that the phase differences across the junctions are same (i.e.,$\delta_1 = \delta_2$) and denoting $I_{total} = I_{2t}$ (i.e., the total current for two Josephson junction), the Eq.(8) implies

$$I_{2t} = 2I_o \sin[\omega_J t + \delta_2]. \quad (9)$$

This is ac Josephson effect for two identical Josephson junction when the phase differences across the junctions are same.

For two non-identical Josephson junctions [i.e. for $I_{01}\#I_{02}$,], the total current equation can be written as

$$I_{total} = \sqrt{\left(I_{01}^2 + I_{02}^2 + 2I_{o1} I_{o2} \cos(\delta_1 - \delta_2)\right)} \times \sin\left[\frac{2eV_o}{\hbar} t + \tan^{-1}\left(\frac{I_{o1} \sin\delta_1 + I_{o2} \sin\delta_2}{I_{o1} \cos\delta_1 + I_{o2} \cos\delta_2}\right)\right] \quad (10)$$

This is *AC Josephson effect for two non-identical Josephson junctions.*

The model describe above can be extended for 3, 4, …N junctions (For details see Ref.5 and references therein). The straightforward result for N junctions is written as

$$I_{Nt} = NI_0 \sin[\omega_J t + \delta_N], \quad (11)$$

when phase differences across the junctions are same $\delta_1 = \delta_2 = \delta_3 = \delta_4 = \delta_N$).

For maximum current the phase and frequency follow the relation [1]:

$$\omega_J t + \delta_N = (4n+1)\frac{\pi}{2}; \quad n = 0,1,2,... \quad (12)$$

Note that in the absence of applied voltage, the equation for two identical Josephson junction [i.e., Eq.(8)] is equivalent to the DC SQUID equation [4-6] provided that

$$\frac{\delta_1 - \delta_2}{2} = \frac{\pi\Phi}{\Phi_0} \text{ and } \frac{\delta_1 + \delta_2}{2} = S_0. \quad (13)$$

Here, S_0 is an adjustable parameter that represents the phase difference ($\delta_1 = \delta_2$) across both the junctions when the magnetic flux $\Phi = n\Phi_0 (\Phi_0 = hc/2e)$ is zero through the junction loop 'ab'.

2.2 Dynamic resistance of Multi- Josephson junction

The concept of dynamic resistance arises from the non-linear voltage current relationship of semiconductor diode. Non linear voltage current relationship is a common phenomenon in Josephson junction. Due to non-linear nature of the I-V curve, there exists a unique value of resistance at every point of the curve which is called dynamic resistance. For the dynamic resistance of multi-Josephson junction, we have derived an equation using the current equation of multi-Josephson junction.

According to Eq. (11), we have the resultant current for N- junctions connected in parallel;

$$I_{Nt} = NI_0 \sin[\omega_J t + \delta_N],$$
$$= NI_0 \sin\left[\frac{4\pi e V_0}{h} t + \delta_N\right], \qquad (14)$$

Where $\omega_J = 2eV_0/\hbar = 4\pi e V_0/h$ has been used.

Now differentiating Eq. (14) w.r.to V_0, we have

$$\frac{dI_{Nt}}{dV_0} = NI_0 \frac{4\pi e t}{h} \cos\left(\frac{4\pi e V_0}{h} t + \delta_N\right),$$
$$= NI_0 \frac{4\pi e t}{h} \sqrt{1 - \sin^2\left(\frac{4\pi e V_0}{h} t + \delta_N\right)},$$
$$= NI_0 \frac{4\pi e t}{h} \sqrt{1 - \frac{I_{Nt}^2}{N^2 I_0^2}},$$
$$= \frac{4\pi e t}{h} \sqrt{N^2 I_0^2 - I_{Nt}^2}. \qquad (15)$$

Let us define the dynamic resistance [see any text book of Physics],

$$R_d = \left(\frac{dI_{Nt}}{dV_0}\right)^{-1}. \qquad (16)$$

Using equation (15) in equation (16), we have

$$R_d = \frac{h}{4\pi N I_0 e t} \frac{1}{\sqrt{1 - \sin^2\left(\frac{4\pi e V_0}{h} t + \delta_N\right)}}. \qquad (17)$$

Where h(=6.63×10^{-34}Js) is the Plank's constant and e=1.6×10^{-19})C, N is the number of Josephson junctions connected in parallel, t is the time in second and I_0 is the maximum current flowing through the junctions.

3. Numerical results

For numerical analysis we have considered N number of identical Josephson junctions connected in parallel with the voltage source V_0. Here we observe the variation of dynamic resistance [using Eq.(17)] with the variable parameters related to it. The variables are the number of Josephson junctions, time and phase differences between the junctions.

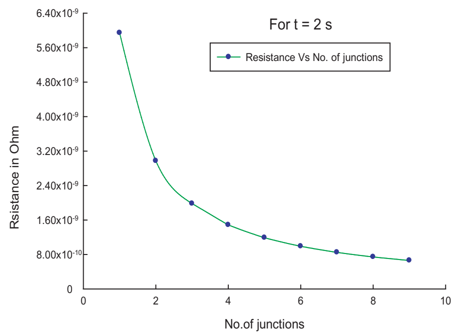

Fig. 2 (a): Variation of dynamic resistance with the number of Josephson junctions for t = 2s.

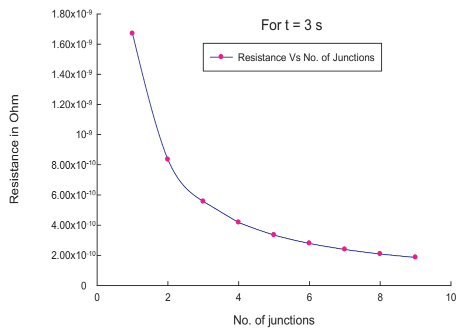

Fig. 2 (b): Variation of dynamic resistance with the Number of Josephson junctions for t = 3s.

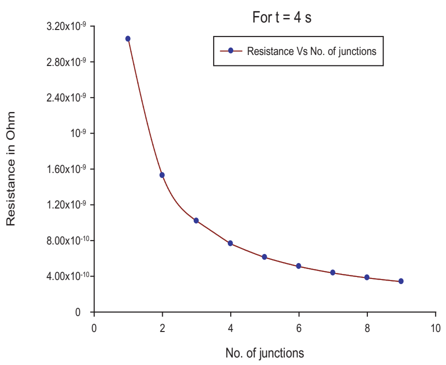

Fig. 2 (c): Variation of dynamic resistance with the No. of Josephson junctions for t = 4s

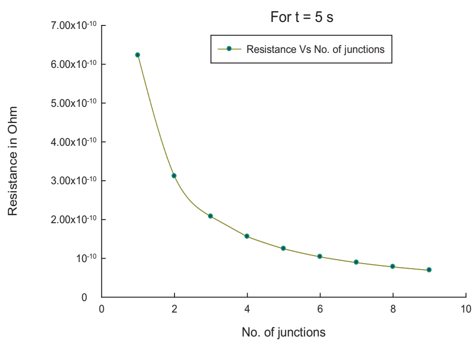

Fig. 2 (d): Variation of dynamic resistance with the No. of Josephson junctions for t = 5s.

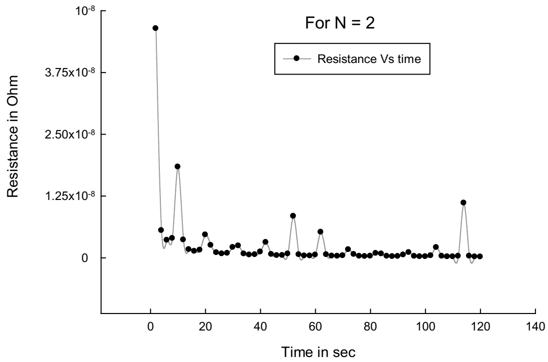

Fig. 3 (a): Variation of dynamic resistance with time for N = 2

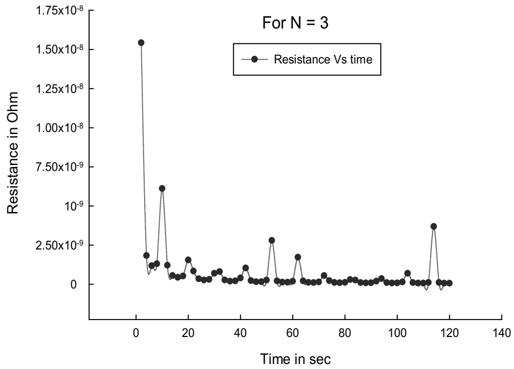

Fig. 3 (b): Variation of dynamic resistance with time for N =3

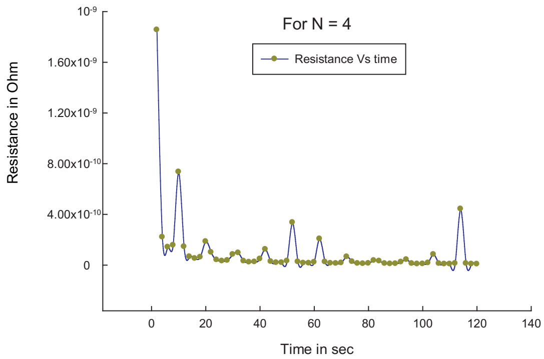

Fig. 3 (c): Variation of dynamic resistance with time for N = 4

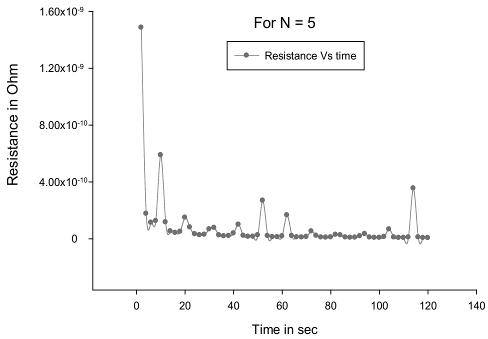

Fig. 3 (d): Variation of dynamic resistance with time for N = 5

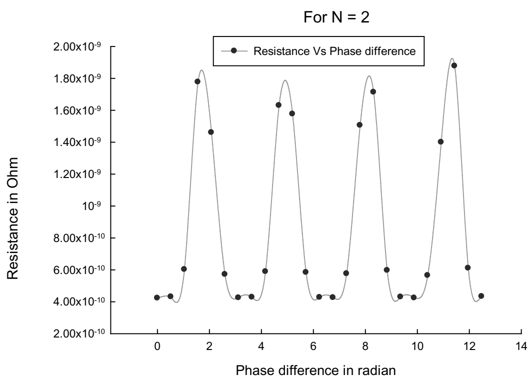

Fig. 4 (a): Variation of dynamic resistance with the phase difference for N = 2

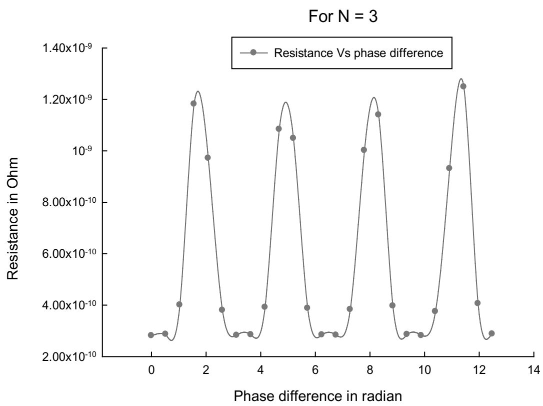

Fig. 4 (b): Variation of dynamic resistance with the phase difference for N = 3

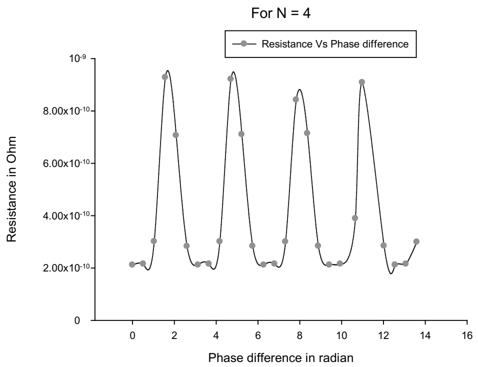

Fig. 4 (c): Variation of dynamic resistance with the phase difference for N = 4

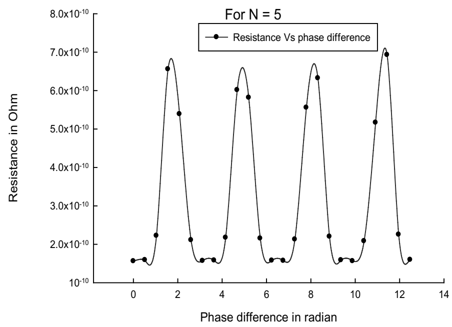

Fig. 4 (d): Variation of dynamic resistance with the phase difference for N = 5

4. Possible Applications of Multi-junction AC Josephson Effect

When proposed the theory of multi-junction ac Josephson effect [5], question raises about its application. Here, we would like to mention a few possible technological applications based on this theory.

4.1 Power amplifier

The multi-junction AC Josephson effect possibly may applicable in a power amplifier [7], as well as a power converter, specially, in SMES (superconducting magnetic energy storage) for electric utilities. For another example, operation of electromagnetic rail launcher (EMRLs) [8] requires very high current (~ 10^3 A-10^4 A) and needs current multiplication. The idea of directly powering an EMRL with a SMES, shown in Figure 6 is under current investigation and seems to be more promising.

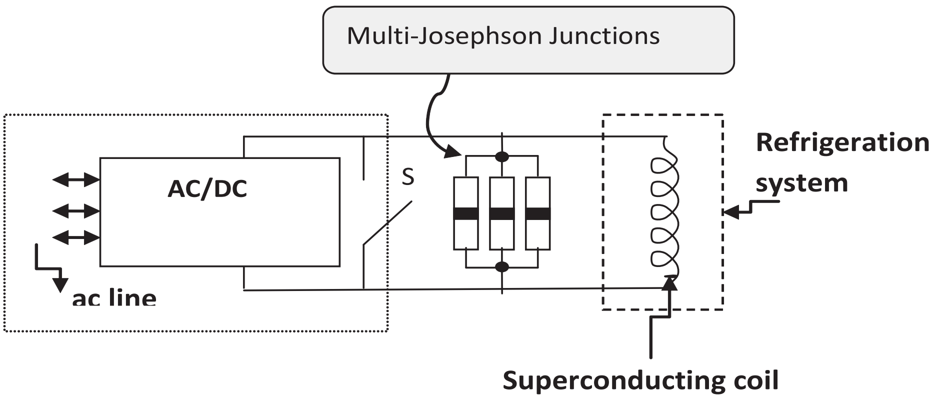

Fig. 5: Proposed [5,7] application of multi-Josephson junction in SMES system.

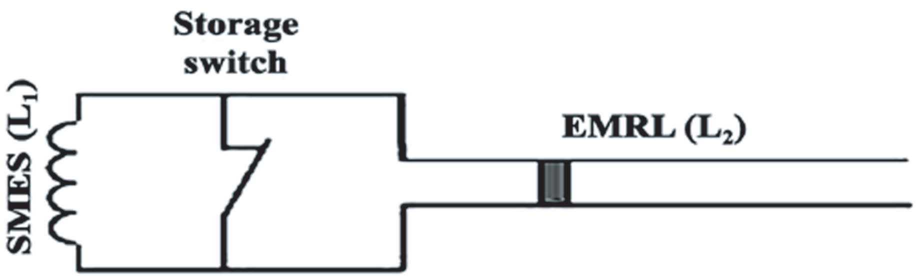

Fig. 6: Improved [5,8] SMES circuit for EMRL powering.

4.2 Terahertz wave source

The artificial generation of electromagnetic waves in the terahertz (THz) frequency range (loosely defined as 0.3-3 THz, 1Tz = 10^{12}Hz) is attracting considerable interest in science and technology. Terahertz sensing and imaging is a rapidly developing technology with wide-ranging applications [9] including security, medicine, quality control and environmental monitoring. But the production and detection of coherent terahertz radiation is still a challenging problem. Addition of multi-Josephson junction may solve the problem in future and improve the devices used in terahertz science and technology.

The AC Josephson effect allows for the construction of voltage to frequency converter that have great potential for THz frequency range. An ac current that appears at the junction oscillates with the Josephson frequency $f_J = 2eV_0 / 2\pi\hbar \, [\because \omega_J = 2\pi f_J]$. According to this equation 1 mV corresponds to an emission frequency of 0.484THz for a single junction. The proportionality between the frequency and the junction voltage is very appealing for the design of frequency-tunable radiation sources.

5. Discussion and Conclusion

From the study of multi junction AC Josephson effect we see that all the junctions operate at the same voltage and produce same frequency. The amount of current increases with the number of the junctions but the frequency remains the same. Hence, we can say that the multi-Josephson junction in parallel connections has large possibility to be synchronized. If it is done the problem of small emissive power can be solved and the multi-Josephson junction may play an important role in sensing and imaging technology.

We have introduced the concept of dynamic resistance in favour of the model of multi Josephson junction proposed earlier. We have studied the dependence of number of junctions, time and phase on the dynamic resistance. The dynamic resistance of a multi-Josephson junction decreases with the increased number of junctions. Hence the amount of current that flowing through the junction increases. The characteristic of decreasing dynamic resistance for parallel combination of the junctions is similar to the parallel combination of normal conductor resistance. Surprisingly, the dynamic resistance of the multi-Josephson junction depends on time. Numerical results indicate that the dynamic resistance changes in a regular pattern. The increased number of junctions do not affects the dependence of time and retain its fixed pattern. Finally, we have concluded that the model of multi-junction AC Josephson effect may play an important role in the development of future devices used in superconductor technology. A multi-Josephson junction is capable of producing enhanced emissive power in terahertz and sub-terahertz frequency ranges, which are still hardly reachable for both electronic and optical devices.

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